THE PARAMETRIC APPROACH TO THE ARCHITECTURAL PROBLEM OF PROJECTING HIGHER-DIMENSIONAL HYPERCUBES TO R^3

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ABSTRACT

This article presents a method for approaching three-dimensional models for n-dimensional hypercubes through polar zonohedra, which – under certain conditions – constitute orthographic isometric projections of such hypercubes. The paper then goes on to present certain sections of the solids in question and the creation of tessellations on the plane. In order to design the zonohedra, use was made of the Rhino program, which combined with the Grasshopper routine allows for the parametric control of the geometric structure of the solid. In other words, it shows – through the proper manipulation of the design algorithm – how zonohedra are produced, constituting projections of higher-dimensional hypercube spaces in three-dimensional space.

Subsequently, the sections of the zonohedra create planar tessellations on the planes, which change depending on how the *n* degree of the zonohedron changes. This results in a table that juxtaposes projections of hypercubes in three-dimensional space and tessellations of a plane, some of which are already known, thus suggesting some sort of correlation between them. This study serves as a formulation of the architectural question surrounding the concept of the projection of polyhedra in general dimension on a plane and suggests an approach involving the parametric control of structures, thus bypassing – to a certain degree – the need for supervision. It also provides an answer to the general question regarding the contemporary role of geometry in the education of architects, which focuses mainly on the

gradual detachment of the architect from the need to constantly monitor the produced form.

From platonic solids to polar zonohedra

The Minkowski sum of n vectors in space is a convex polyhedron with n(n-1) faces, where n is the number of the different directions of the vectors. If the vectors are equal in size, then the faces of the convex polyhedron will be shaped as rhombi and the polyhedron will constitute an equilateral zonohedron. Equilateral zonohedra are considered as 3-dimensional projections of n-dimensional hypercubes. The more symmetrised the initial vectors are, the more symmetrical the resulting zonohedra will be.

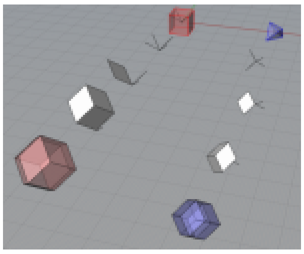

Fig. 1

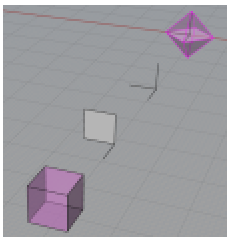

Fig. 2

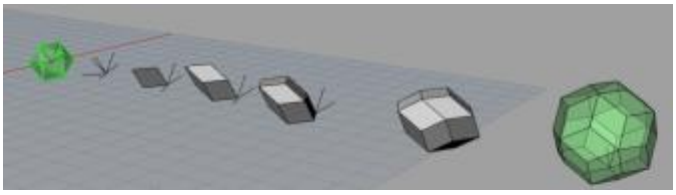

Fig. 3

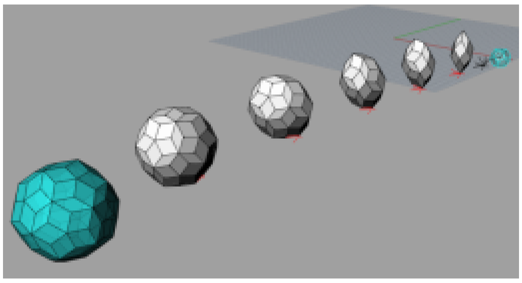

Fig. 4

The most symmetric zonohedra are those resulting from platonic solids, with the vectors directed towards the linear segments, which project the vertices of each polyhedron from its centre. Thus, the cube and regular tetrahedron result in a rhombic dodecahedron (Fig. 1), the regular octahedron results in a cube (Fig. 2), the regular icosahedron results in Kepler's golden rhombic triacontahedron (Fig. 3), whose faces are rhombi with a diagonal ratio equal to the golden ratio Φ, and the regular dodecahedron results in rhombic enneacontahedron (Fig. 4), whose faces consist of two types of rhombi (60 of the one type and 30 of the other). The above zonohedra constitute 3D models of hypercubes.

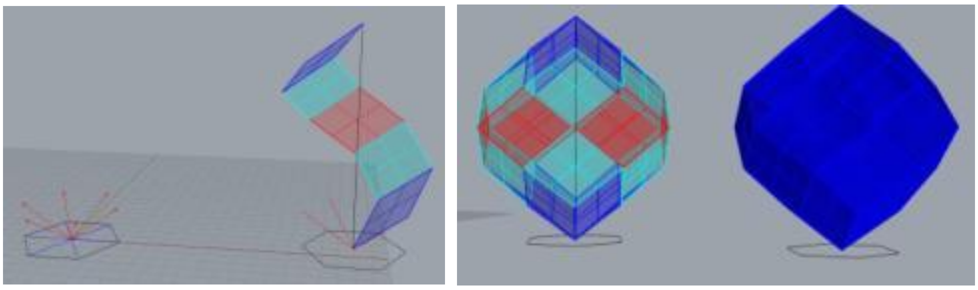

Fig. 5

Polar zonohedra form a unique category of zonohedra. Let us take a regular n-gon in the plane and line segments ε_1, ε_2,..ε_v, which connect the centre O of the polygon to its vertices. Then let us take the equal vectors δ_1, δ_2,…δ_v, with O being the common starting point, which are projected in the plane of the n-gon by ε_1, ε_2,…ε_v. The zonohedron resulting from the Minkowski sum of vectors δ_1, δ_2,…δ_v, which is called a polar zonohedron, is a convex polyhedron whose n-fold axis is the vertical line in the centre of the plane of the polygon with n(n-1) rhomboid faces laid out in zones around the axis (Fig. 5). The algorithm allows us to create polar zonohedra by controlling the number of vectors, their inclination to the plane of each polygon and their size. This results in a variety of forms. When several of the sides of a regular polygon are infinite, then the zonohedron leans towards a surface of revolution. It has been observed that when the angle of inclination of the vectors to the plane of the polygon is 36.264°, the n-polar zonohedron is considered a 3D orthographic isometric projection of the n-hypercube.

The transition from smaller to larger dimensions

Considering that the cube can create a spatial tessellation, which – with the appropriate sections – can result in planar tessellations, we will explore the possibility of creating spatial structures and, by extension, planar tessellations from hypercubes by working with their three-dimensional models.

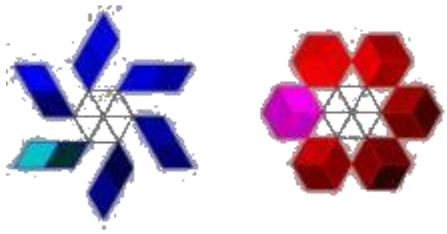

Fig. 6

This exploration is also based on the fact that each n-polar zonohedron and, consequently, each n-hypercube can result from the composition of zonohedra of a lower order.

Thus, for example, the **rhombic triacontahedron** (which is a 3D model of the 6-cube) can result as follows: Let us take the parallelohedra defined by two triads of the six vectors that determine the rhombic triacontahedron. The combination of two oblong and two oblate parallelohedra (Fig. 6) results in a rhombic dodecahedron (3D model of the 4-cube) (Fig. 7). This, along with the use of six additional parallelohedra, results in a rhombic icosahedron (3D model of the 5-cube) (Fig. 8). The rhombic icosahedron combined with 10 parallelohedra, results in a rhombic triacontahedron (which is a 3D model of the 6-cube) (Fig. 9).

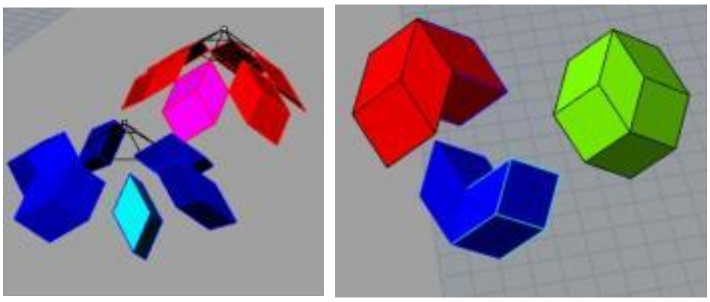

Fig. 7

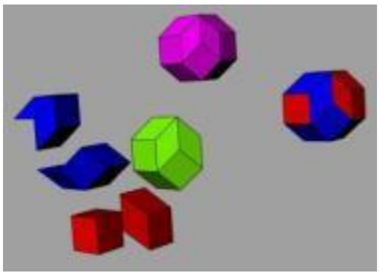

Fig. 8

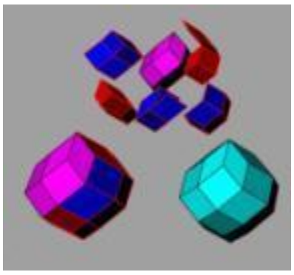

Fig. 9